(8 pages) **Reg. No. :** ....

Code No. : 5764 **Sub. Code : WMAE 22**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2024.

Second Semester

Mathematics

Elective III — MATHEMATICAL STATISTICS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. A function which assigns to each element $c \in \mathcal{C}$ one and only one real number $X(c) = x$ is called a ________ variable.
   (a) real (b) complex
   (c) random (d) constant

2. The value of $\Pr(S)$ is ________ where S is the sample space.
   (a) 0 (b) 8
   (c) 1 (d) 4

3. The moment generating function $M(t)$ is defined by ________.
   (a) e^{tx} (b) $E\left(e^{tx}\right)$
   (c) $E(xf(x))$ (d) $E(X)$

4. The mean of a binomial distribution having m.g.f. as $(.5 + .5e^t)^7$ is ________.
   (a) 7/2 (b) 2.6
   (c) 3.5 (d) 7/5

5. The value of the conditional probability $P(A \cap B)$ is ________ if A and B are independent events.
   (a) $P(A).P(B)$ (b) $P(A) + P(B)$
   (c) $P(B)/P(A)$ (d) 1

6. The random variables X_1 and X_2 are said to be stochastically independent if and only if $ff(x_1, x_2) =$ ________.
   (a) $f_1(x_1)$ (b) $f_1(x_1)f_2(x_2)$
   (c) $f_2(x_2)$ (d) $f_1(x_2)$

7. If $(1-2t)^{-6}$, $t < 1/2$ is the moment generating function of a random variable then its variance is ________.
   (a) 3 (b) 12
   (c) 24 (d) 5

8. The formula for $\overline{X}$ is ________.
   (a) $\frac{\Sigma X_i}{2n}$ (b) $\frac{\Sigma X_i}{n}$
   (c) $\frac{\Sigma X_i}{4n}$ (d) $\frac{\Sigma x X_i}{n}$

9. The m.g.f. of a normal distribution is $e^{3t + \frac{36t^2}{2}}$ then the standard deviation is
   (a) 4 (b) 6
   (c) 1 (d) 3

10. If F have an F distribution with parameters r_1 and r_2 then $1/F$ has an F distribution with parameters ________.
    (a) r_1/r_2 (b) $r_1.r_2$
    (c) r_2 and r_1 (d) $1/r_2$

11. The variance S^2 of n random variables $X_1, X_2, ...X_n$ is ________.
    (a) $\sum_{i=1}^{n}(X_i - \overline{X})^2/n$ (b) $\sum_{i=1}^{n}(X_i - \overline{X})$
    (c) $\sum_{i=1}^{n}(X_i - \overline{X})^3/n$ (d) $\sum_{i=1}^{n}(X_i + \overline{X})$

12. Determine the constant c so that $f(x) = cx(1-x)^3, 0 < x < 1$, 0 elsewhere for the beta distribution.
    (a) 1 (b) 9
    (c) 20 (d) 4

13. If $\lim_{n \to \infty} F_n(y) = F(y)$ for every point y then the random variable Y_n is said to have a ________ distribution with distribution function $F(y)$.
    (a) one to one (b) cauchy
    (c) limiting (d) continuous

14. A distribution function of discrete type which has a probability of 1 at a single point is called as ________ distribution.
    (a) inventory (b) elements
    (c) cube (d) degenerate

15. The limiting distribution of a random variable is degenerate then the random variable is said to be ________ to the constant that has the probability of 1.
    (a) converge stochastically
    (b) diverge stochastically
    (c) both (a) and (b)
    (d) neither (a) nor (b)

[P.T.O.]

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let X denote the random variable with $E(X)=3$ and $E(X^2)=13$ then find the lower bound for $\Pr(-2<X<8)$ using Chebyshev's inequality.

Or

(b) Let X have the p.d.f. $f(x)=\frac{1}{2}(x+1)$, $-1<x<1$, 0 elsewhere. Find the mean and variance of X.

17. (a) Derive the m.g.f. of Binomial distribution and hence find the mean and variance of the distribution.

Or

(b) Let X_1 and X_2 have the joint p.d.f. $f(x_1, x_2)=2$, $0<x_1<x_2<1$. Find the conditional p.d.f. of X_1 given $X_2=x_2$.

18. (a) If $(1-2t)^{-6}$, $t<1/2$ is the moment generating function of the random variable X then find $\Pr(X<5.23)$.

Or

(b) Let X be $\chi^2(10)$. Find $\Pr(3.25 \le X \le 20.5)$. Find a if $\Pr(a<x)=0.05$ and $\Pr(X \le a)=0.95$.

19. (a) Let $\overline{X}$ be the mean of the random sample of size 25 from a distribution that is $n(75, 100)$. Find $pr(71<\overline{X}<79)$.

Or

(b) Let F have an F distribution with parameters r_1 and r_2. Prove that $1/F$ has an F distribution with parameters r_2 and r_1.

20. (a) Let Y_n denote the nth order statistic of a random variable from the uniform distribution with $f(x)=1/\theta$, $0<x<\theta$, $0<\theta<\infty$ else. Prove that $Z_n = n(\theta - Y_n)$ has a limiting distribution with distribution function $G(z)$.

Or

(b) Let Z_n be $\chi^2(n)$. The m.g.f. of Z_n is $(1-2t)^{-n/2}$, $t<1/2$. Investigate the limiting distribution of the random variable $Y_n = (Z_n - n)/\sqrt{2n}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

21. (a) Let X have the p.d.f. $f(x)=x+2/18$, $-2<x<4$, 0 elsewhere. Find $E(X+2)^3$ and $E(6X-2(X+2)^3)$.

Or

(b) State and prove Chebyshev's inequality.

22. (a) Let X_1 and X_2 have the joint p.d.f. $f(x_1, x_2)=\frac{x_1+x_2}{21}$, $x_1=1, 2, 3$, $x_2=1, 2, 0$, elsewhere. Find the marginal p.d.f. of X_1 and X_2 hence find $\Pr(X_1=3)$ and $\Pr(X_2=2)$.

Or

(b) Let the random variables X_1 and X_2 have the joint p.d.f. $f(x_1, x_2)$. Then prove that X_1 and X_2 are stochastically independent if and only if $f(x_1, x_2)$ can be written as a product of a non negative function of x_1 along and a non negative function of x_2 alone.

23. (a) Derive the moment generating function of the normal distribution.

Or

(b) If the random variable X is $n(\mu, \sigma^2)$, $\sigma^2>0$ then prove that $V=(x-\mu^2)/\sigma^2$ is $\chi^2(1)$.

24. (a) Derive t distribution.

Or

(b) Let Y_1, Y_2, Y_3 be the order statistics of a random sample of size 3 from a distribution having p.d.f. $f(x)=1$, $0<x<1$, 0 elsewhere. find the p.d.f. of $Z_1 = Y_3 - Y_1$.

25. (a) State and prove Central limit theorem.

Or

(b) Let $F_n(y)$ denote the distribution function of a random variable Y_n whose distribution depends on the positive integer n. Let c denote a constant which does not depend upon n. Prove that the random variables Y_n converges stochastically to the constant c if and only if for every $\varepsilon>0$ $\lim_{n\to\infty} \Pr(|y_n - c|<\varepsilon)=1$.

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